Forced Vibration Analysis of a Micro End Mill Cutter by Mode Superposition Method with Damping Included

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Abstract

Machine tool structures are multi – degree – of – freedom systems and, as such, cannot easily be expressed mathematically. However, their dynamic characteristics strongly influence machine tool chatter. When a tool is engaged upon a steady cut, there is no dynamic excitation and static stiffness only of the machine tool structure is important. When the tool contacts some irregularity in the work surface, e.g. a hard spot, it will bounce or vibrate relative to the workpiece. This sort of forced vibration of tool is very significant when it comes to machine a workpart on a Miniaturized Machine Tool (MMT). This is due to the fact that tool tip dynamics of a micro tool is an important area of research study so far as micromachining is concerned. Furthermore, catastrophic micro tool failure is rampant in micro cutting because of slenderness nature of the micro tool. Moreover, frequent tool failures will result in machine tool breakdowns and hence affect productivity and surface topography of the finished workpart. In view of this fact, the present work focuses on forced vibration analysis of a micro end mill cutter by including damping effect too in the study of its dynamic performance. The damping characteristics of the micro end mill cutter can be represented appropriately using proportional damping. This assumption of proportional damping is adequate in mode superposition analysis of a linear dynamic system, for instance, a micro end mill as in present case. For qualitative dynamic analysis/study of a micro endmill cutter, linear model works well. In the present analysis, it is assumed that the micro end mill cutter is a cantilever beam, rigidly supported by the spindle holder (via collet) and the end mill vibrates due to constant cutting force (generally below 10N in all directions) applied to the cutter. The physical model of the cutter (that is flute length or axial cutting length) is idealized by the finite element approach (FEA) discretizing the same into suitable number of finite beam elements for the purpose of analyzing its dynamic impact on the overall performance of the micro cutting.

Keywords: Micro end mill cutter, Forced vibration analysis, Proportional damping, Cantilever beam model, Finite element approach.

1. INTRODUCTION

Micromilling which is defined as the downscaling of the conventional milling process involving the use of end mill diameters in the sub – millimeter range has become an established process for manufacturing of three – dimensional meso and micro components in metals and alloys. In micromilling, forces are exerted between the tool and the workpiece [1]. Furthermore, tools used in micro milling are characterized by long and smooth connections between the shaft and the cutting region. Therefore, one of the fundamental contributions to the accuracy of milled component is the deflection of the tool due to the cutting forces. Forced vibration is due to the tooth passing frequency. Thus, if the natural frequency of the machine, tool, or workpiece is near the characteristic frequency of the cutting force, a high dynamic amplification happens [2, 3].

In the cutting process, vibration is a dynamically unstable phenomenon. All machines vibrate and as the state of the machine worsens (imbalance of the spindle, or other important shaft, defect of bearing and spindle) the vibration level increases. While the increase in machine vibration allows us to detect a defect, the analysis of machine vibration characteristics allows us to identify its cause. The terms rigidity and stability are interrelated to each other [4].

Dynamic rigidity is one of the most critical characteristics of machine tools especially for high precision and high performance applications. It determines the dynamic response of the machine structure to cutting forces and inertial loads during the acceleration and deceleration of the axes. High amplitude vibrations in response to these loads may result in poor machined part quality and potential damage to the machine. A machine tool dynamic's rigidity depends on many factors such as its configuration (geometry), size and construction method, etc. The overall dynamic rigidity in a machining system depends on all of the components involved, i.e., machine tool, tooling, fixtures, workpiece etc. Therefore, the rigidity of all components in a machining system is critical as the one with the lowest rigidity usually determines the rigidity of the whole system. And therefore, the present work focuses on dynamic analysis of a micro end mill cutter this is owing to the fact that micro tool is the one which will exhibit lowest rigidity of all the machine tool elements [5].

The modal analysis theory for an undamped multi – degree – of – freedom (MDOF) is applicable for dynamic structures when damping is negligible. The presence of damping does not change every aspect of the theory presented for undamped systems. However, more mathematical treatment is needed in order to extend the modal analysis theory into the case for a damped multi – degree – of freedom system. The two main damping models used in MDOF modal analysis are the viscous damping model and the structural damping model. Like mass and stiffness properties, now the distribution of damping is an important property as well as its amount. The coupled equations of motion (EOM) for an undamped MDOF system can be uncoupled using the principle of orthogonality. Therefore, analysis of individual modes becomes convenient. However, once damping is present, it is generally difficult or not possible to uncouple the EOM. Therefore, damped MDOF systems demand extra theoretical treatment [6].

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2. MODAL ANALYSIS OF A LINEAR VIBRATING SYSTEM

Modal analysis is a procedure by which the orthogonality properties of the modes of vibration are utilized to transform the equations of motion (EOM) from a physical coordinate system to a principal coordinate system where the EOM are uncoupled. Rather than solving the $n-$original simultaneous ordinary differential equations (ODEs), each of the decoupled equations representing single oscillators can be solved independently. This procedure is valid for undamped systems as well as certain damped systems known as proportionally damped systems. The reason such a principal coordinate system exists is because the modes have certain orthogonlaity with respect to the mass and stiffness matrixes. The assumption of proportional viscous damping is made quite frequently many times only implicitly.

The normal mode method, or modal analysis, applies only to undamped systems or systems for which the damping matrix can be made mathematically equivalent to a combination of the mass and stiffness matrices. Damping plays an insignificant role in the steady – state response of a periodically forced system if the forcing frequency is not near one of the system resonances. For special systems where the damping matrix is linearly related to the mass and stiffness matrices, that is proportional damping case, the simultaneous diagonalization of the stiffness and mass matrices can be accomplished along with that of the damping matrix [7].

The analysis of a $n-$ DOF damped systems deserves special attention, as one formula, mimicking the single – DOF case, which is possible for undamped systems, is not possible here. To this end, we start by converting the system of $n-$second order differential equations into a system of $2n$ first order differential equations. We thus express the frequency equation, in normal form by recalling the usual transformation.

2.1 Proportional Damping Model

A proportional damping model is the first analytical model used to study damping for an MDOF system. Unlike mass and stiffness properties, damping cannot usually be modelled. This became a stumbling block to the analysis of a damped MDoF system. The proposition of proportional damping enabled the analysis to proceed. Proportional damping has found significant applications in finite element analysis (FEA) where damping needs to be incorporated in order to carry out meaningful response analysis and prediction. In modal analysis theory, the significance of proportional damping will become evident when we realize that a system with proportional damping would have the mode shapes identical to that of its undamped counterpart. We begin with the analysis of free vibration. If the damping distribution of the system of $n-$ DOF with viscous damping is denoted as a matrix $\mathbf{C}$, the matrix equation of motion of system (free vibration) is given by

$$\mathbf{M\ddot{x}} + \mathbf{C\dot{x}} + \mathbf{Kx} = \mathbf{0} \tag{1}$$

If it is forced vibration analysis, the above EOM can easily be modified by inducting forcing term to the right side of the Eq. (1) as follows, i.e.,

$$\mathbf{M\ddot{x}} + \mathbf{C\dot{x}} + \mathbf{Kx} = \mathbf{f} \tag{2}$$

Here, matrix $\mathbf{C}$ is positive definite or positive semi – definite. Unlike the undamped case, there generally does not exist a set of principal coordinates, which uncouple equation (1). In particular, if we use the mode shape matrix $\mathbf{X}$, then both the mass matrix $\mathbf{M}$ and the stiffness matrix $\mathbf{K}$ can be diagonalized. However, damping matrix $\mathbf{C}$ cannot be leaving the equations still uncoupled. The exceptional damping distribution which does allow the diagonalization of matrix $\mathbf{C}$ as well as matrix $\mathbf{K}$ and $\mathbf{M}$ is called proportional damping. Rayleigh indicated that if the viscous damping matrix $\mathbf{C}$ is proportional to mass and stiffness matrices (or that if the damping forces are proportional to the kinetic and potential energies of the system), then it can be written as [4]:

$$\mathbf{C} = a\mathbf{M} + b\mathbf{K} \tag{3}$$

where a and b are real positive constants. Equation (1) can then be uncoupled like the matrix equation for an undamped system. The substitution of Eq. (3) into Eq. (1) leads to:

$$\mathbf{M\ddot{x}} + (a\mathbf{M} + b\mathbf{K})\mathbf{\dot{x}} + \mathbf{Kx} = \mathbf{0} \tag{4}$$

Repeating the uncoupling process for the undamped case using the undamped mode shape function matrix $\mathbf{X}$ (obtained by assuming $\mathbf{C} = \mathbf{0}$ from Eq. (1)) will lead to the uncoupled equations:

$$[\,^{\cdot\cdot}m_{r_{\cdot\cdot}}]\,\ddot{\boldsymbol{\alpha}} + [\,^{\cdot\cdot}c_{r_{\cdot\cdot}}]\dot{\boldsymbol{\alpha}} + [\,^{\cdot\cdot}k_{r_{\cdot\cdot}}]\boldsymbol{\alpha} = \mathbf{0} \tag{5}$$

where diagonal matrix $[\,^{\cdot\cdot}c_r]$ is called the modal damping matrix or generalized damping matrix of the system.

Premultiplying the Eq. (2) by $[\mathbf{X}]^{\mathrm{T}}$, and taking note of coordinate transformation from physical to normal coordinate system, $\mathbf{x} = \mathbf{X\alpha(t)}$ and Eq. (5), we obtain

$$\ddot{\boldsymbol{\alpha}}_{\mathbf{i}} + \mathbf{C}\dot{\boldsymbol{\alpha}}_{\mathbf{i}} + [\boldsymbol{\omega}^{2}]\boldsymbol{\alpha}_{\mathbf{i}} = \bar{\mathbf{f}}_{\mathbf{i}} \tag{6}$$

where $\boldsymbol{\omega}^2 = \mathrm{diag}\left[\omega_i^2\right]$, $i = 1,\ldots\ldots,n$ is the matrix of the eigenvalues and the modal forces $\bar{\mathbf{f}}_{\mathbf{i}}(t)$ are

$$\bar{\mathbf{f}}_{\mathbf{i}}(t) = \frac{\mathbf{f}_{\mathbf{i}}}{\bar{\mathbf{M}}_{\mathbf{i}}} = \frac{\mathbf{X}_{\mathbf{i}}^{\mathbf{T}}\mathbf{f}}{\mathbf{X}_{\mathbf{i}}^{\mathbf{T}}\mathbf{M}\mathbf{X}_{\mathbf{i}}}$$

If the proportional damping of $\mathbf{C} = a[\mathbf{M}] + b[\mathbf{K}]$ is used, $\mathbf{C}$ is the diagonal matrix.

$$\mathbf{C} = a[\mathbf{I}] + b[\boldsymbol{\omega}^{2}] \tag{7}$$

Obviously the undamped mode shape matrix $\mathbf{X}$ can diagonalize the promotional damping matrix as well as the mass and stiffness matrices. Therefore, $\mathbf{X}$ (which is a real matrix) is also the mode shape matrix for the system having proportional viscous damping model. For modal analysis, this is the most important characteristic from a proportional damping. Eq. (4) consists of $n-$uncoupled equations. Using the theory of an SDOF system, the damped natural frequency of the r^{th} mode, $\underline{\omega}_r$, can be estimated by

$$\underline{\omega}_r = \omega_r\sqrt{1-\zeta_r^2} \tag{8}$$

$$\zeta_r = \frac{a}{2\omega_r} + \frac{b\omega_r}{2} \tag{9}$$

Like an single degree – of – freedom (SDOF) system, ζ_r is defined as the damping ratio. The difference is that this time damping is for r^{th} mode. Eq. (8) shows that the damping ratio for a system with proportional damping is different for each mode. The proportional damping (viscous damping) as defined in Eq. (3) shall be expressed in a more general form as follows [8]:

$$\mathbf{K}\mathbf{M}^{-1}\mathbf{C} = \mathbf{C}\mathbf{M}^{-1}\mathbf{K} \qquad (10)$$

3. MODELLING OF A MICRO END MILL CUTTER

3.1 Micro End Mill Cutter

The main drawbacks lie in the tool, which is critical in terms of size, wear and precision. Unlike conventional milling, in micomilling the main source of the compliance is the tool itself, which can account for up to 80 – 90 % of the total compliance at the tool tip. In micromachining, the edge radius of the tool tends to be the same order – of – magnitude as the chip thickness. The concept of a minimum chip thickness, below which no chip will form, or minimum depth of cut below which no material removal occur, has been investigated by a few researchers [3].

The features of large shank, taper and reduced diameter at the cutting edges as shown in Fig. 1 are unique characteristics of a micro end mill. This design is required for micro end mills to provide tool holding with the shank and to keep the length to diameter ratio small in the fluted region for stiffness.

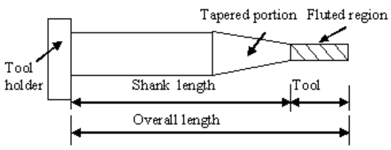

Fig. 1. Micro end mill [5]

3.2 Finite Element Modelling of a Micro End Mill Cutter

The end mill cutter is idealized as a cantilever beam (see Fig. 2) based on Euler – Bernoulli beam theory in which normal stresses are much higher than shear stresses such that normal stress is the principal stress. Fig. 2 depicts the micro end mill details and Fig. 3 illustrates FE idealization of the cutter.

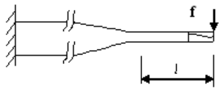

Fig. 2. Cantilever beam model for a micro end mill

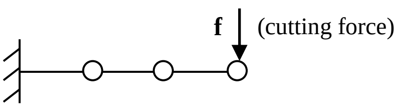

Fig. 3. FE idealization of micro end mill cutter

The detailed FEA of a micro end mill cutter which is modelled as a simple undamped forced vibration oscillator (cantilever beam model) is given in [8]. As for damping matrix is concerned, a complete procedure of deducing the same is highlighted in [8]. The mass, stiffness, and damping matrices of a micro end mill cutter are thus given as detailed below:

$$\mathbf{K} = 10^{8}\begin{bmatrix} 3.5343 & 0.0000 & -1.7671 & 0.0004 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & -0.0004 & 0.0000 & 0.0000 & 0.0000 \\ -1.7671 & -0.0004 & 3.5343 & 0.0000 & -1.7671 & 0.0004 \\ 0.0004 & 0.0000 & 0.0000 & 0.0000 & -0.0004 & 0.0000 \\ 0.0000 & 0.0000 & -1.7671 & -0.0004 & 1.7671 & -0.0004 \\ 0.0000 & 0.0000 & 0.0004 & 0.0000 & -0.0004 & 0.0000 \end{bmatrix}$$

$$\mathbf{M} = 10^{-5}\begin{bmatrix} 0.1094 & 0.0000 & 0.0189 & -0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & -0.0000 & 0.0000 & 0.0000 \\ 0.0189 & 0.0000 & 0.1094 & 0.0000 & 0.0189 & -0.0000 \\ -0.0000 & -0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.0000 \\ 0.0000 & 0.0000 & 0.0189 & 0.0000 & 0.0547 & -0.0000 \\ 0.0000 & 0.0000 & -0.0000 & -0.0000 & -0.0000 & 0.0000 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 0.1170 & 0.0000 & 0.0202 & -0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & -0.0000 & 0.0000 & 0.0000 \\ 0.0202 & 0.0000 & 0.1170 & 0.0000 & 0.0202 & -0.0000 \\ -0.0000 & -0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.0000 \\ 0.0000 & 0.0000 & 0.0202 & 0.0000 & 0.0585 & -0.0000 \\ 0.0000 & 0.0000 & -0.0000 & -0.0000 & -0.0000 & 0.0000 \end{bmatrix}$$

By substituting the above matrices into Eq. (10), one can easily observe that the equality in Eq. (10) is satisfied. So the proportional damping model as suggested in this work would work well for a linear structure like micro end mill cutter. And the forced response of a micro end mill cutter with damping included can be estimated by means of Duhamel integral (considering only steady – state response) as follows [4]:

$$x_i(t) = \frac{1}{\underline{\omega}_r}\int_0^t g_i(\tau)e^{-\zeta_i\omega_i(t-\tau)}\sin\underline{\omega}_r(t-\tau)d\tau \qquad (11)$$

with damping frequency, $\underline{\omega}_r = \omega_r\sqrt{1-\zeta_r^2}$

Decoupling of EOM via normal coordinates forms the crux of mode superposition method. To say explicitly, the whole system's dynamic response is now represented as an individual or single – DOF response corresponding to the respective natural frequencies by the above Duhamel integral.

4. DYNAMIC RESPOSNE OF A MICRO END MILL CUTTER

4.1 Dynamic Response in Principal Coordinates

Mode superposition analysis is the efficient tool for the evaluation of the response of the linear systems subjected to dynamic agencies. Two well – known mode superposition methods are available in the literature, the mode displacement method and the mode acceleration method. In the present analysis, the use of mode (displacement) superposition method is described in detail in the context of dynamic analysis of a micro end mill cutter. Mode superposition is basically a transformation technique that will change the mode of operation of dynamic system from one coordinate frame to another coordinate frame in an affable manner. That is to say that, the coupled equations of motion (EOM) will be uncoupled by principal coordinates or normal coordinates through orthogonality properties of modes of vibration. A brief procedure of mode (superposition) displacement method is given hereunder:

Orthogonality of the mode shapes implies

$$\mathbf{X^T M X = I}\,;\ \mathbf{X^T K X = \Omega} = \text{diag}\{\omega_1^2.....\omega_n^2\} \tag{11}$$

If the viscous damping is proportional, then

$$\mathbf{X^T C X = Z} = \text{diag}\{2\zeta_1\omega_1, 2\zeta_2\omega_2....., 2\zeta_n\omega_n\}$$

$\mathbf{X}_i$ = modal matrix; $\mathbf{X}_i = [\mathbf{X_1}, \mathbf{X_2}, ------, \mathbf{X_n}]$

i^{th} column is $\mathbf{X}_i$

Relationship between physical coordinates ($\mathbf{x}$) and principal (normal) coordinates ($\boldsymbol{\alpha}$) is

$\boldsymbol{\alpha} = \mathbf{X^{-1} x}$ or $\mathbf{x} = \mathbf{X}\boldsymbol{\alpha}$

EOM in physical coordinates and normal coordinates is now given as follows [9]:

$$\mathbf{M\ddot{x}} + \mathbf{C\dot{x}} + \mathbf{Kx} = \mathbf{f}$$

$$\ddot{\boldsymbol{\alpha}} + \mathbf{Z}\dot{\boldsymbol{\alpha}} + \boldsymbol{\Omega\alpha} = \mathbf{G(t)} \tag{12}$$

where

$\mathbf{X^T C X = Z} = \text{diag}\{2\zeta_1\omega_1, 2\zeta_2\omega_2....., 2\zeta_n\omega_n\}$,

$\mathbf{X^T K X = \Omega} = \text{diag}\{\omega_1^2.....\omega_n^2\}$ and $\mathbf{G(t)} = \mathbf{X^T F}$

The above equation is called $n-$ second order uncoupled equations of motion (EOM) and of the form,

$\ddot{\alpha}_i + 2\zeta_i\omega_i\dot{\alpha}_i + \omega_i^2\alpha_i = G_i(t)$, $i = 1,....,n$

The solution of which is given by

$$\alpha_i(t) = \frac{1}{\omega_i\sqrt{1-\zeta_i^2}}\int_0^t e^{-\zeta_i\omega_i(t-\tau)}\sin\underline{\omega}_i\sqrt{1-\zeta_i^2}(t-\tau)G_i(\tau)d\tau \tag{13}$$

By convolution integral, Eq. (13) can be recast into

$$\alpha_i(t) = \frac{G_i}{\omega_i^2}\left[1-e^{-\zeta_i\omega_i t}\left\{\frac{\zeta_i}{\sqrt{1-\zeta_i^2}}\sin\underline{\omega}_i t + \cos\underline{\omega}_i t\right\}\right] \text{for } i = 1,....,n$$

with damping ratios $\zeta_1 = 0.04311$ $\zeta_2 = 0.0069$ $\zeta_3 = 0.0024$

Table 1

Natural frequencies and Damped frequencies of a micro end mill cutter [8]

Mode No.	Natural Frequencies (rad/sec)	Damped Frequencies (rad/sec)
1	$\omega_1 = 1.235\times10^6$	$\underline{\omega}_1 = 1.234\times10^6$
2	$\omega_2 = 7.744\times10^6$	$\underline{\omega}_2 = 7.744\times10^6$
3	$\omega_3 = 21.717\times10^6$	$\underline{\omega}_3 = 21.717\times10^6$

Substituting the above numerical values (as listed in Table 1) for natural and damping frequencies in Eq. (13), we get the dynamic response (for the first three modes) in principal or normal coordinates as follows and further it is to be noted that the estimated damping ratios for the dynamic model of micro end mill cutter are less than 0.05.Due to this fact the difference between natural and damping frequencies is almost negligible (see Eq. (11)).

$$\alpha_1(t) = 1.4824\times10^{-6}F\left[1-e^{-\zeta_1\omega_1 t}\left\{\frac{\zeta_1}{\sqrt{1-\zeta_1^2}}\sin\underline{\omega}_1 t + \cos\underline{\omega}_1 t\right\}\right]$$

$$\alpha_2(t) = -6.323\times10^{-8}F\left[1-e^{-\zeta_2\omega_2 t}\left\{\frac{\zeta_2}{\sqrt{1-\zeta_2^2}}\sin\underline{\omega}_2 t + \cos\underline{\omega}_2 t\right\}\right]$$

$$\alpha_3(t) = -6.64172\times10^{-9}F\left[1-e^{-\zeta_3\omega_3 t}\left\{\frac{\zeta_3}{\sqrt{1-\zeta_3^2}}\sin\underline{\omega}_3 t + \cos\underline{\omega}_3 t\right\}\right]$$

Further simplification of the above equations will yield,

$$\alpha_1(t) = F\left[1-e^{-53240.85t}\left\{0.04315\sin(1.234\times10^6 t) + \cos(1.234\times10^6 t)\right\}\right]$$

$$\alpha_2(t) = F\left[e^{-53433.6t}\left\{0.0069\sin(7.744\times10^6 t) + \cos(7.744\times10^6 t)\right\} - 1\right]$$

$$\alpha_3(t) = F\left[e^{-52120.8t}\left\{0.0024\sin(21.717\times10^6 t) + \cos(21.717\times10^6 t)\right\} - 1\right]$$

And for a cutting force of 10 N, the above will become

$$\alpha_1(t) = \left[10-10e^{-53240.85t}\left\{0.04315\sin(1.234\times10^6 t) + \cos(1.234\times10^6 t)\right\}\right]$$

$$\alpha_2(t) = \left[10e^{-53433.6t}\left\{0.0069\sin(7.744\times10^6 t) + \cos(7.744\times10^6 t)\right\} - 10\right]$$

$$\alpha_3(t) = \left[10e^{-52120.8t}\left\{0.0024\sin(21.717\times10^6 t) + \cos(21.717\times10^6 t)\right\} - 10\right]$$

5 CONCLUSIONS

In this research work, an attempt has been made to model a micro end mill cutter by treating it as a simple damped forced oscillator. And the dynamic response in principal (normal) coordinates has been determined utilizing Duhamel integral. In determining the dynamic response of the cutter, the EOM expressed in physical coordinates were decoupled by means of natural or principal coordinates. This sort of analysis is called modal (displacement) superposition analysis. The beam elements were used to develop a FE model of the micro end mill cutter and the relevant equations were formulated based on the proportional damping models. In addition, miniature tools have vey high natural frequencies which are difficult to excite and measure. Therefore, numerical prediction methods as presented in this work were well suited to determine tool tip dynamics. The study of dynamic aspects of a micro tool assumes significance from the viewpoint of visualizing its dynamic behaviour under stipulated forced cutting conditions.

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