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Specifying the Manufacturing Deviation of 3D Reference Axes in Special Cases

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Abstract

In many precision machines and equipments, there are two or three reference axes intersecting at a point theoretically as per the drawing. One example is gyro spin axes construction. The other examples are N/C machine tool spindle axis and table axis as reference axes. Yet another example is the three bevel gear reference axes of space equipment like helicopter, intersecting at a common point. In all the above examples, theoretically the axes have to meet at a common point. But at manufacturing stage one has to specify the acceptable deviation. Presently each axis is measured separately for its straightness or perpendicularity with some surface. Though the axes are measured separately one cannot say to what extent they are meeting or how closely they are approaching to the theoretical intersection point. Further there are no standards or methods to specify this error.

The author has earlier established and published different ways of fitting axis. Some six methods of specifying the deviation/error were also defined. During the processes it is found that there is a need of simplifying the way of specifying the error which is a combination of most of the six defined error specifications. This is called Truncated Cylindrical Deviation (TCD). In this the error is specified by two parameters of the cylinder (diameter and height), such that the three axes are tangential to the cylinder. The three tangents when projected on to a plane perpendicular to the axis form a triangle. The highest and lowest tangent points determine the height of the cylinder. When the reference axis coincides with the one of the tangents to the cylinder, the height of the truncated cylinder is infinite and when the tangents are projected on to a plane perpendicular to the axis only two lines and a point are formed. In such cases the two tangents points on the tangent lines determine the height of the truncated cylinder. A method of evaluating such deviation which is a special case requiring a different algorithm is discussed in this paper

Keywords: Evaluation of reference axes, Error evaluation, geometric dimensioning and tolerancing.

1. INTRODUCTION

There are many applications in metrology or manufacturing, for which there are no specified methods or standards for measurement and evaluation or to specify the manufacturing tolerances. Generally for metrology application three mutually perpendicular coordinate planes are used as reference planes for measurements. But there are cases where mutually perpendicular axes are taken as reference in NC machine tools, Gyro spin axis. Further in case of a helicopter gear box with three bevel gears axis, which are not mutually perpendicular are supposed to but pass through a common point theoretically. But one has to specify on the drawing the manufacturing tolerance as how closely the axis are expected for proper functioning.

In this context there is a need for suitable evaluation techniques for application in 3D tolerancing. Presently the trend is complete elimination of 2D drawings and migration to 3D with dimensioning and tolerancing in 3D drawings. In this regard a new ASME Y14.41-2003, STANDARD FOR CAD in to the digital domain is already available [1] Method for evaluation of two mutually perpendicular axes has been reported [2], which evaluates an artefact with number of holes along two perpendicular rows for its hole positioning accuracy and alignment of the holes. A method for evaluation of three mutually perpendicular/non-perpendicular & intersecting axes has been reported [3, 4], which evaluates the three likely intersecting axes of precision equipment.

After developing different methods for evaluation of the axes supposed to passing through a point, six different ways of specifying the deviation are presented [5 to 10]. Among these six different specifications one of the ways of specifying the error namely Tangential Spherical Deviation (TSD) has lead another concept as Truncated Cylindrical Deviation (TCD)[11]. This specification is simple. In this specification all the three axes will be tangential to largest possible imaginary cylinder whose axis is in the direction of viewing. As each axis will touch the cylinder as a tangent, there are three such tangent points. When the cylinder is truncated at the highest and lowest tangential points, the height of the cylinder and the diameter of the truncated cylinder indicate the total error. When the viewing direction is parallel to one of the axis, special cases arises, which is explained in this paper.

2. PRESENT PROBLEM DEFINITION (SPECIAL CASE)

The actual problem faced and the work done so far and the special cases are explained with the help of three figures. Fig.1 shows a Cartesian coordinate system with three likely intersecting axes and the measurements (taken using a flexible CMM called ROMER) corresponding to the three axes of an equipment like helicopter gear box. The problem is: how to find-out the likely point of intersection of these axes and how closely they are approaching and how to evaluate and specify the deviations/error? This has been solved by specifying a viewing direction and a projection plane perpendicular to the viewing direction, to project the axes on to the plane. This results in three lines on the projected plane forming a triangle. The diameter of the in-circle of the triangle so formed is called TSD as in Fig.2. The circle so obtained is actually the projection of a cylinder. If we locate the actual tangential points on the cylinder corresponding to the tangential points of the in-circle and the triangle formed by the projection of the 3 lines,

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we get the dimensions of the truncated cylinder [11]. This paper presents an algorithm for determining the dimensions of the truncated cylinder in special cases when the viewing direction is parallel to one of the axes as in Fig.3, the projected tangent lines are OA, OB and point P

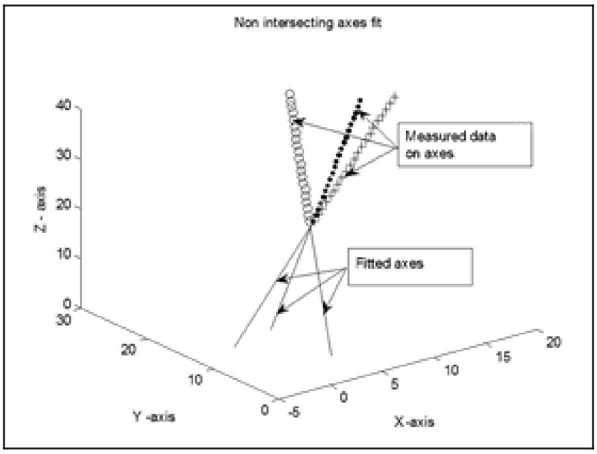

Fig. 1. Reference coordinate system along with the three axes to be evaluated.

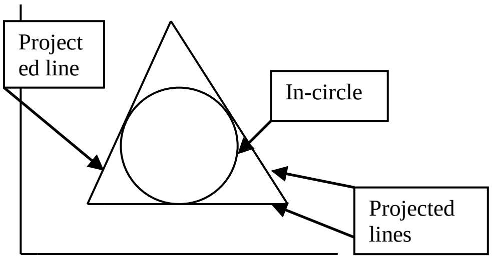

Fig.2. Axes projected in the plane perpendicular to the specified mean axes/viewing direction.

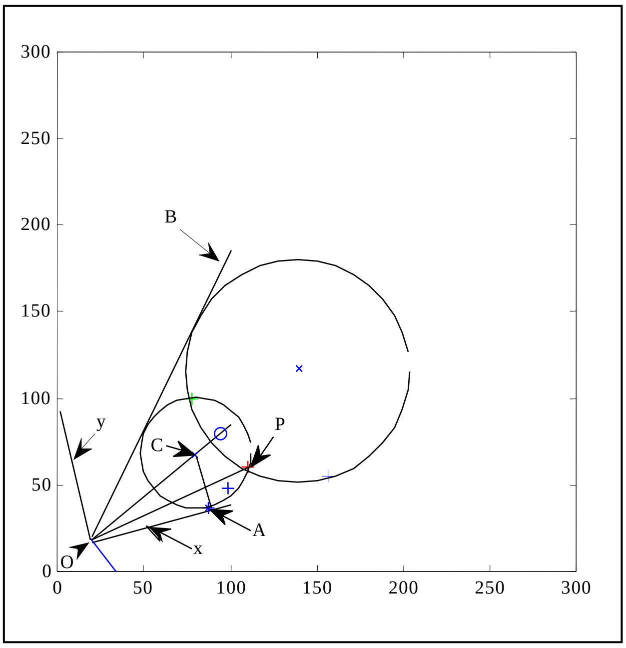

Fig.3. Projected axes (OA, OB and point P) with local coordinate system (Special cases).

3. MATHEMATICAL ANALYSIS

Now in the special cases (Fig.3) there are two lines and a point. Now to find TSD or TCD, we need to construct a circle (cylinder in 3D) tangent to two lines and passing through a point (line/generator of cylinder in 3D). We generally feel that that there is only one circle which is tangent to two lines and passing through a point. But from Fig.3 we can find two circles satisfying the above condition (namely tangent, tangent, point). Obviously we need the diameter of the smaller circle to specify TSD. An extensive step by step procedure to arrive at the radius of the circle has been presented Murthy [9]. Also an alternative simple method is presented [10] as explained below which is required to find tangent locations to compute the height of truncated cylinder.

Let θ be the angle between the projected intersecting lines OA and OB as shown in Fig.3. Here point A is also marked as the tangent point. Let α be the angle between OA and OP, where P is the projection of one line as a point P. Let C be the centre of the circle (projected axis of cylinder) such that radius CP =CA in the projected plane with local coordinate system with O as origin. Considering the suffixes c and p for coordinate locations of C and P, we have:

$$CP^2 = CA^2$$

i.e $$(x_c - x_p)^2 + (y_c - y_p)^2 = y_c^{\,2} \qquad (1)$$

Also as the angle between OA and OB is θ, the angle between OA and OC is θ/2.

Also $\tan\left(\frac{\theta}{2}\right) = {}^{y_c}/_{x_c}$ = k (a constant & slope)

Or $$y_c = k\,x_c \qquad (2)$$

Substituting equation (2) in equation (1) and solving the quadratic equation for x_c we get

$$x_c = (x_p + k\,y_p) \pm \sqrt{\{(k^2-1)y_p^2 + 2\,k\,x_p y_p\}} \qquad (3)$$

As α is the angle between lines OA and OP

$$x_p = OP\cos(\alpha) \qquad (4)$$

$$y_p = OP\sin(\alpha) \qquad (5)$$

The angles θ and α are computed from the dot product relation of the 2D or 3D vectors of those intersecting lines namely OA, OB, and OP. Thus equation (3) gives two solutions for the circle centre. The line OA and its extension (projection of line 1) contains (x_1, y_1, z_1) and (x_2, y_2, z_2) (Fig.3, Fig.A1, Fig.A2). The point A (Fig.3) divides the points (x_1, y_1, z_1) and (x_2, y_2, z_2) (two points on line OA) externally or internally in a ratio p:q. Similarly the actual tangent contact point on the cylinder (whose projection is the circle with centre C) is found in the same ratio p:q. between the points (a_1, b_1, c_1) and (a_2. b_2, c_2)., thus giving the coordinates of the contact point. Similarly the actual contact point of the other tangent corresponding to line OB is obtained. From these two contact points on the cylinder, the height of the truncated cylinder is determined and given in Table.1

The mathematical procedures involved in the procedure explained above are given below: (Fig.A1 in Appendix A)
Line 1 in space is defined by 2 points (a_1, b_1, c_1) and (a_2, b_2, c_2)
Line 2 in space is defined by 2 points (a_3, b_3, c_3) and (a_4, b_4, c_4)
Line 3 in space is defined by 2 points (a_5, b_5, c_5) and (a_6, b_6, c_6)
The above data are given in Table1 as first 6 rows.
The direction cosines of the three lines computed from line data are given in the next three rows
The direction cosines along the actual view direction are given in the next rows along with the TSD, TCD values in the following two rows. Now this view direction is replaced by the direction cosines of lines 1, 2, and 3 one at a time in the special case discussed in this paper and the results are given in the last three rows.

The coordinates of the projected points of (a_i, b_i, c_i) on to the plane (passing through origin) perpendicular to the view direction are represented as (x_i, y_i, z_i) and their computation given in [11]. This plane contains two lines (OA, OB) and the third line as point (P). The point of intersection of lines OA and OB is represented (X_{10}, Y_{10}) and all the projected points (2 points on each line) are transferred to local coordinates system in the plane with (X_{10}, Y_{10}) as origin and line OA or OB as X axis. In Fig.3 OA is assumed as X-axis. Then the values of x_c and y_c , is the centre of the circle where y_c is also the radius of the circle are given from equations (4) and (5) Then the direction AC (in Fig.3) or opposite direction (CA) is the Y-axis in the local coordinate system.
A visual representation of the points on the projected plane at different stages of computation along with the fitted circle with centre (x_c , y_c) and radius y_c as in Fig.3 will aid in confirming the computed values. From equation (3), there are two values of x_c and hence two values of y_c. We select the lowest value of y_c.
In the absence of visual/graphical representation, the following procedure will be used

1) Compute distances d_i (i=1 to 6) for all 6 points from local coordinate system (two of the distances will be same as one line is parallel to view direction and is represented as a point).
2) Compute angles made by each line with local X axis
3) Arranging the angles in ascending order along with α and θ will fix the location of all points in local coordinate system
4) Since x_c is the length of the tangent point (tangent to circle) from local coordinate system, we need to find the corresponding tangent point on the space tangent (tangent to cylinder)
5) Considering line 1 (having points 1 and 2 on it) and line 2 having points 3 and 4 on it, and line 3 having points 5 and 6 on it, and Considering D_1 to D_6 as the distances from (X_{10}, Y_{10} , Z_{10}) to all the 4 points in space lines and considering d_1 to d_6 as the distances from in the same point (X_{10} , Y_{10}) to the projected points, the height location tangent points on space line are given by

   $h_1=D_1-(D_4-D_3)*(d_3-x_{c})/(d_4-d_3)$;
   $h2=D_5-(D_6-D_5)*(d_5-x_c)/(d_6-d_5)$;

   and height of truncated cylinder is

   $h=abs(h_1-h_2)$

In Table 1 last three rows indicate the size of truncated cylinder (diameter and height) when viewing reference axis along each of the 3 axis directions.

TABLE 1 SUMMARY OF RESULTS

Description	a	b	c
Line1 Start Point	7.45	7.49	6.58
Line 1 End Point	108.06	202.40	304.75
Line 2 Start Point	7.66	7.98	6.59
Line 2 End Point	206.39	106.51	309.56
Line 3 Start Point	8.21	8.13	7.15
Line 3 End Point	302.06	304.75	106.58
Direction cosines of	l	m	n
Line1	0.27	0.53	0.81
Line 2	0.53	0.26	0.81
Line 3	0.68	0.69	0.23
(View Direction)	0.53	0.53	0.66
TSD	0.052		
TCD	ϕ=0.052 and h=1.125		
Results (truncated cylinder diameter ϕ and height h) when view direction parallel to line1, line2 or line3.			
View direction	TCD (diameter ϕ=, height h=)		
Line1	ϕ=0.1698, h=0.3717		
Line2	ϕ=0.3869, h=0.1638		
Line3	ϕ=0.3872, h=1.5586		

4. RESULTS AND CONCLUSIONS

Table 1 shows the summary of results for the 3D reference axis evaluated as a truncated cylinder for error when the viewing direction is along line1, line 2 and line 3. Though 6 different ways of specifying the error are indicated, the Truncated Cylindrical Deviation (TCD) seems to be more appropriate as volumetric error. Thus the solution is unique and does not require minimum zone values like in evaluation of sphericity [12], circularity, flatness, cylindricity [13] etc.

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APPENDIX – A (Projection of points on to a plane)

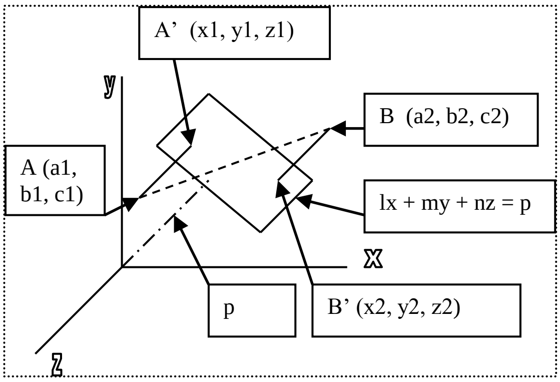

Fig.A1. Axes AB projected as A'B' in the plane perpendicular to specified axis (plane given by lx + my + nz = p)

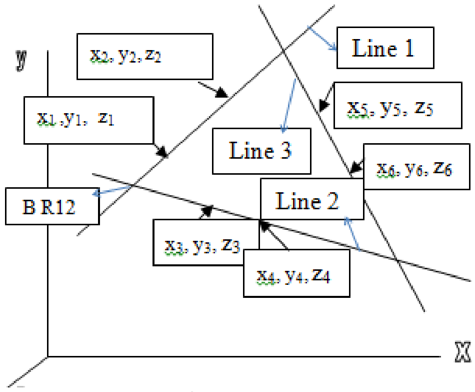

Fig.A2. Projection of 3D lines on to a plane, which are intersecting to form a triangle. In the special case only two lines intersect (line 1 and line 2) and third line (line 3) will be a point(P) as in Fig.3.