Proceedings of 10th International Conference on Precision, Meso, Micro and Nano Engineering (COPEN 10)
December 07 – 09, 2017, Indian Institute of Technology Madras, Chennai – 600 036 INDIA

Isoparametric Lines-based Sampling for Measurement of Freeform Surfaces

G. Rajamohan*

Department of Manufacturing Engineering
National Institute of Foundry and Forge Technology, Ranchi - 834 003, INDIA

Abstract

Freeform surfaces are generally defined in parametric space using B-Splines or NURBS. Their measurement using continuous scanning coordinate measuring machines (CMMs) may be performed along either or both parametric directions, based on some suitable sampling strategy. The sampling strategy, which includes the number of isoparametric scan lines and their positions on freeform surfaces, affects the measurement time and accuracy. Considering scanning along one parametric direction only, the present work proposes an approach to determine the most appropriate scanning direction that gives better accuracy than other scanning direction for the same number of scan lines. The proposed approach uses the complexity of freeform surfaces, calculated using mean curvature and chord length. The developed approach is validated using the uniform and stratified uniform isoparametric sampling methods based on few suitable examples.

Keywords: Coordinate measuring machines, isoparametric lines, sampling strategy.

1 INTRODUCTION

Manufactured parts must fulfil the dimensional and geometrical specifications to satisfactorily perform their intended functions. The inherent machine tool errors and approximations involved in the computer-aided design and manufacturing (CAD/CAM) of parts with freeform surfaces lead to geometrical deviations in manufactured parts, making their measurement and verification inevitable. Their verification requires large amounts of data that may be obtained using contact or non-contact type coordinate measuring machines (CMMs), which operate in either discrete or continuous scanning mode. Sampling for the measurement of freeform surfaces is a challenging task due to their complex nature. For continuous scanning, strategies such as iso-planar sampling, isoparametric sampling, etc. are in practical use. Regardless of the strategy employed, the scan lines must be distributed over the entire freeform surface thereby ensuring that the measurement is adequate. A short review of research relevant to sampling for discrete type CMMs has been provided first for information purposes, followed by those for continuous type CMMs.

Despite being inadequate, measuring instruments generally use the geometry-independent sampling strategies, such as uniform sampling [1-6]. Geometrical properties of the freeform surfaces, such as mean and Gaussian curvatures, have been used to get non-uniform distribution of sampling points [3-8]. Curvature-based methods may also result in inadequate measurement as they distribute the sampling points at higher curvature regions. A hybrid distribution with user-defined weights for uniform and curvature-dependent sampling has been proposed to avoid the accumulation of sample points at higher curvature regions [3-4]. Other geometry-based sampling strategies, such as the chord length, maximum allowed distance between any two adjacent points and number of sample points per knot span [9], uniform arc length [10] and surface area [11] have also been reported. Combined use of more than one geometric characteristic, for example, patch size and mean or Gaussian curvature [5-6, 8] and mean curvature and surface area [11], have also been tried. Above strategies usually distribute a given number of sample points. Instead, adaptive sampling can be performed to find the number of sample points by repeating the sampling until certain user defined criterion is satisfied. One of the earliest adaptive sampling methods is based on the surface normal curvature and the interpolating curves between initially sampled points [12]. Obeidat and Raman [8] used maximum and minimum Gaussian curvatures and patch sizes to derive different adaptive sampling strategies at patch levels. These algorithms have been verified later using milled freeform surfaces [13]. An adaptive sampling method that obtains sampling points by dividing the spline mass equally has been shown to yield higher precision [14]. Yu et al. proposed an adaptive sampling method based on form error models obtained by superimposing the appropriate form errors on nominal surfaces [15].

For continuous scanning CMMs, ElKott and Veldhuis proposed two adaptive isoparametric sampling strategies that start with a minimum number of lines equal to the order of the surface and add more sample lines at those locations where the positional deviation (automatic sampling) or curvature-change (curvature-change based sampling) is maximum [16]. Zhou et al. proposed a search-based adaptive sampling strategy using the absolute curvature to identify the locations of isoparametric sample lines automatically [17]. Rajamohan and Shunmugam proposed a dominant isoparametric lines based sampling strategy to locate the given number of isoparametric lines [18]. The dominant isoparametric lines are identified using the surface shape index. This approach has been modified into an adaptive sampling method that starts with an assumed initial sample size [19]. The sampled isoparametric lines are used to construct a reference surface and the positional deviations are computed. Sampling is continued by the addition of a new sampling line every time until the specified sampling accuracy or maximum specified sample size is reached.

Measurement of freeform surfaces using continuous scanning CMMs can be done along one or both parametric directions, i.e. u and v. The present work proposes an approach to decide the direction for performing the scanning based on the complexity of freeform surfaces that will yield better accuracy for an equal number of scan lines. Surface complexity is calculated using the curvature and chord length. The developed approach is tested using the uniform isoparametric sampling method based on few suitable examples.

* Author to whom correspondence should be made, Email: grajamohan.nifft@gov.in

ISBN: 978-93-80689-28-9

Understanding of the CAD representation of freeform surfaces using B-splines/Non-uniform Rational B-splines (NURBS) [20] and that of computing the mean curvature (H) at any point on the surface are assumed [21-22]. Rest of the paper is organized as follows. Section 2 describes about the methodology used in this work, Section 3 describes about the proposed approach to determine the apt parametric direction for sampling and Section 4 briefs about the sampling strategies used. The results obtained using some example freeform surfaces are discussed in Section 5, followed by the conclusions in Section 6.

2 METHODOLOGY

Sampling strategy for freeform surfaces using the continuous scanning CMMs is concerned with establishing the number of isoparametric scan lines (sample size) and their locations so as to ensure the measurement adequacy. Positional deviations are used to quantify the measurement adequacy. While the number of isoparametric scan lines may be fixed or obtained adaptively by satisfying certain criteria, locating them requires a suitable strategy meeting the desired adequacy. In the present work, the sampling strategy starts with discretization of design freeform surface using a user-defined fine spacing (Δu, Δv) in parametric space. Both the values have been uniformly taken as 0.005. The mean curvatures are then computed at discrete points obtained as intersections of u and v isoparametric lines. Sample size (N_s) is randomly taken as 12, while the minimum sample size should be equal to the number of control points along that parametric direction. Both u and v isometric sampling plans are generated with this sample size.

The isoparametric lines obtained using the appropriate sampling strategy are used to obtain a reference surface for the freeform surface using skinning approach [20, 24]. Individual reference surfaces are constructed based on the respective isoparametric sample lines. The positional deviations between the original and reference surfaces are computed as the normal distance between all the points obtained as intersections of u and v isoparametric lines and the reference surface. The sum of absolute values of maximum and minimum positional deviations is used to verify: (a) the efficacy of proposed sampling direction determination method, and (b) the measurement adequacy of sampling method used.

3 DETERMINATION OF SAMPLING DIRECTION

Arc length and chord length are synonymously used throughout the paper. Combined mean curvature and chord length of the isoparametric lines is used to compute the complexity of given freeform surface along any parametric direction. Assuming N isoparametric lines in any parametric direction, Eq. (1) can be used to compute N values of complexity indices. Their mean is then computed.

$$\lambda_i = \beta K_i + (1-\beta) L_i ;\ \ 0 \le \beta \le 1 \qquad (1)$$

where, β is the weight and K_i and L_i are the curvature and arc length approximants respectively of i^{th} isoparametric line [23]. Similar calculations are then carried out for the other parametric direction also. The parametric direction with higher mean value is chosen for sampling, as higher mean implies more severe the surface variations along that direction than the other. The value of β is taken to be 0.5 such that the curvature and arc length are given equal importance.

4 SAMPLING STRATEGIES USED

Sampling strategy used plays a crucial role on the measurement results. This paper uses two sampling strategies, viz. uniform and stratified uniform, for the purposes stated earlier. While the uniform sampling strategy is commonly used, stratified uniform sampling is proposed in this paper as improvement to uniform sampling. A brief description of both these strategies are given below.

4.1 Uniform sampling

Uniform sampling method distributes the u isoparametric scan lines using Eq. (2).

$$u_i^* = u_{\min} + (i-1)\frac{u_{\max} - u_{\min}}{(N_s - 1)} \qquad (2)$$

The calculated u_i^* positions are aligned with discretized grid by selecting u_i values that are closer to them. Eq. (2) can be used to calculate the sample positions for v isoparametric lines also, by replacing u with v.

4.2 Stratified uniform sampling

In this sampling method, the desired number of isoparametric lines are shared among different columns of the surface based on their complexity. A column denotes the set of isoparametric lines between two successive knots. Assume a surface with two columns along u parametric direction and let their complexities be λ_{c1} and λ_{c2} respectively. A total of three isoparametric lines will be reserved first, i.e. those corresponding to u_{min}, u_{max} and one at the knot position. Remaining (N_s - 3) lines will be shared between the two columns as shown in Eq. (3).

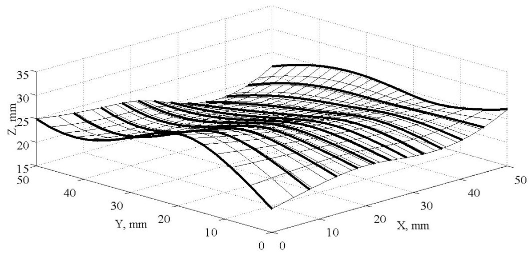
$$N_{c1} = \frac{\lambda_{c1}}{(\lambda_{c1} + \lambda_{c2})}(N_s - 3);\ \ N_{c2} = N_s - N_{c1} - 3 \qquad (3)$$

where, N_{c1} and N_{c2} are respectively the sample lines shared by columns 1 and 2. Within a column, the isoparametric scan lines are distributed using the uniform sampling method.

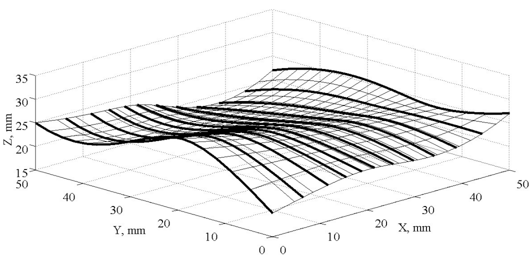
5 RESULTS AND DISCUSSION

Sampling strategies considered in this paper have been coded in C++ using Visual Studio 2015 run on an Intel i5 processor (64-bit) powered PC running Windows 10 Pro, aided by 4GB RAM. Reference freeform surfaces were constructed by skinning [22, 26] and positional deviations were calculated at 201×201 points (resulting from the assumed 0.005 spacing in parametric space that varies from 0 to 1) as the shortest distance between these points and the reference surface. It lies along the normal vector to reference surface from respective point. The sum of absolute values of the maximum (p_{max}) and minimum (p_{min}) positional deviations were used to compare the sampling strategies. The strategy that yields minimum value of the sum is considered to be more effective.

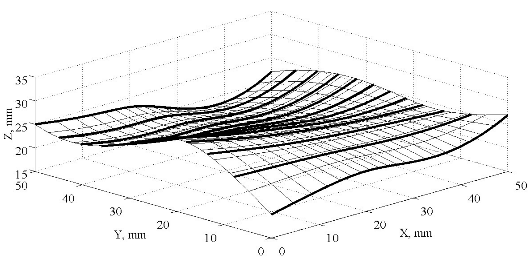
Seven bi-cubic example freeform surfaces (S_1 to S_7) of 50 mm × 50 mm size and of varying complexities have been used for testing the proposed approach for determination of sampling direction and for comparing the sampling strategies. Figure 1 shows the example surface S_4 superimposed with the identified isoparametric lines. The thick lines are sampled isoparametric lines and thin lines are the (non-sampled) isoparametric lines, shown for ease of visualizing the freeform surface. Figures 1(a) and 1(c) show the isoparametric lines obtained using uniform *u* and *v* sampling plans respectively, while 1(b) and 1(d) show the isoparametric lines obtained using the stratified uniform *u* and *v* sampling plans respectively.



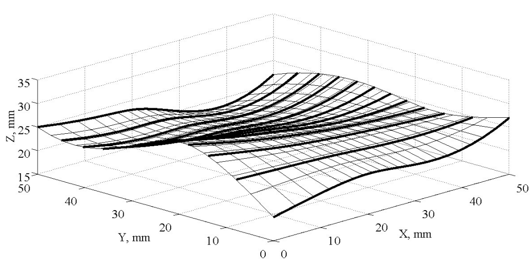
(a) Uniform u sampling plan



(b) Stratified uniform u sampling plan



(c) Uniform v sampling plan



(d) Stratified uniform v sampling plan

Figure 1 Results of different sampling plans for N_s = 12

Table 1 shows the results obtained using the example freeform surfaces. Second column represents the determined sampling direction for the respective freeform surface. It is implied that either the uniform sampling strategy or the stratified uniform sampling strategy corresponding to the determined direction should show the minimum sum of absolute values of positional deviations. The two values in columns 3 to 6, one positive and one negative, indicate the maximum and minimum positional deviations obtained using the respective sampling strategies.

The surfaces S_1 and S_2 are single patch surfaces, specified with 4×4 control points and hence the stratified sampling cannot be applied to them. The positional deviations corresponding to the stratified sampling methods are therefore shown as dashes. For surface S_1, the determined direction of *u* results in a positional deviation of 0.260 μm (shown in boldface) while *v* sampling results in a positional deviation of 1.603 μm. In the same manner, the positional deviations shown in boldface lead to the lowest sum of absolute values for that particular example surface. Surfaces S_3 to S_5 have 5×5 control points and S_6 to S_7 have 6×6 control points. It can be clearly seen that for all the example freeform surfaces, the determined sampling direction has resulted in the lowest sum of absolute values of positional deviations. Hence, the proposed approach for finding the apt parametric direction for sampling is effective.

Comparison between uniform and stratified uniform sampling strategies can also be done in terms of sum of absolute values of positional deviations. It is generally expected that the stratified uniform sampling should result in the lowest sum. This is due to the fact that the stratified uniform sampling strategy provides higher share of isoparametric lines to more complex columns along a sampling direction. This aspect when combined with the determined sampling direction implies that the stratified sampling strategy corresponding to that direction should yield the minimum absolute sum. This condition is met with example surfaces S_3, S_4, S_5 and S_7. In case of surface S_6, the uniform sampling strategy yields a slightly smaller sum of 42.296 μm against the sum of 42.871 μm yielded by the stratified uniform sampling method. With just about 0.5 μm difference, it can be stated that the stratified uniform sampling strategy generally results in a better sampling plan when compared to the uniform sampling strategy.

Table 1 Sampling direction and positional deviations (μm)

Surf.	Dir.	*u* Sampling Plans		*v* Sampling Plans	
		Uniform	Stratified uniform	Uniform	Stratified uniform
S_1	*u*	+0.016 -0.244	---	+1.164 -0.439	---
S_2	*v*	+8.220 -6.803	---	+2.776 -3.214	---
S_3	*u*	+0.000 -0.000	+0.000 -0.000	+2.342 -9.524	+1.841 -8.662
S_4	*u*	+7.635 -11.049	+3.661 -5.747	+22.590 -36.667	+24.520 -39.012
S_5	*v*	+23.480 -58.940	+9.958 -31.572	+5.712 -5.278	+4.751 -2.591
S_6	*v*	+18.884 -54.724	+18.978 -52.116	+14.947 -27.349	+17.264 -25.607
S_7	*u*	+28.127 -27.964	+24.928 -28.815	+17.306 -37.612	+26.350 -36.907

6 CONCLUSIONS

Certain isoparametric sampling plans (the *u* or *v* sampling plan) work well with certain surfaces, which becomes apparent only after analyzing the results. The present research is based on the firm belief that it must be possible to identify the right sampling direction that best suits for any freeform surface by analyzing it before sampling. In this direction, a methodology based on the complexity analysis of the freeform surface has been proposed.

Based on the validation using suitable examples of freeform surfaces, it has been found that the positional errors computed along the direction determined by the proposed method always resulted in smaller values. Thus, the proposed method can be used to obtain the most appropriate sampling direction so that better measurements can be carried out in less time. A stratified uniform sampling method has been proposed. The examples used reveal that stratification leads to better measurement plan in contrast with uniform sampling strategy for the same sample size.

References

[1] Wang, J., X. Jiang and L. A. Blunt, A survey of potential sampling strategies for measurement of structured surfaces, *Computing and Engg. Annual Researcher's Conference* (*CEARC '09*). University of Huddersfield, Huddersfield, 184-189.

[2] Duffie, N., J. Bollinger, R. Piper and M. Kroneberg, CAD directed inspection and error analysis using patch databases, *Annals of the CIRP*, **33** (1984) 347-350.

[3] Pahk, H. J., Y. H. Kim, Y. S. Hong and S. G. Kim, Development of computer-aided inspection system with CMM for integrated mould manufacturing, *Annals of the CIRP*, **42** (1993) 557-560.

[4] Pahk, H. J., M. Y. Jung, S. W. Hwang, Y. H. Kim, Y. S. Hong and S. G. Kim, Integrated precision inspection system for manufacturing of moulds having CAD defined features, *Int. Journal of Advanced Manufacturing Technology*, **10** (1995) 198-207.

[5] ElKott, D. F., H. A. ElMaraghy and A. O. Nassef, Sampling for freeform surfaces inspection planning, *ASME Design Engineering Technical Conf.*, Nevada, (1999) 1-9.

[6] ElKott, D. F., H. A. ElMaraghy and W. H. ElMaraghy, Automatic sampling for CMM inspection planning of freeform surfaces, *Int. Journal of Production Research*, **40** (2002) 2653-2676.

[7] Cho, M. W. and K. Kim, New inspection planning strategy for sculptured surfaces using coordinate measuring machine, *Int. Journal of Production Research*, **33** (1995) 427-444.

[8] Obeidat, S. M. and S. Raman, An intelligent sampling method for inspecting freeform surfaces, *Int. Journal of Advanced Manufacturing Technology*, **40** (2009) 1125-1136.

[9] Ainsworth, I., M. Ristic and D. Brujic, CAD-based measurement path planning for freeform shapes using contact probes, *Int. Journal of Advanced Manufacturing Technology*, **16** (2000) 23-31.

[10] Rajamohan, G., M. S. Shunmugam and G. L. Samuel, Effect of probe size and measurement strategies on assessment of freeform profile errors using coordinate measuring machine, *Measurement*, **44** (2011) 832-841.

[11] Rajamohan, G., M. S. Shunmugam and G. L. Samuel, Practical measurement strategies for verification of freeform surfaces using coordinate measuring machine, *Metrology & Measurement Systems*, **XVIII** (2011) 209-222.

[12] Edgeworth, R. and R. G. Wilhelm, Adaptive sampling for coordinate metrology, *Precision Engineering*, **23** (1999) 144-154.

[13] Obeidat, S. M., R. H. Fouad and N. Mandahawi, Verification of new sampling methods on small scale freeform surfaces, *Jordan Journal of Mechanical and Industrial Engineering*, **6** (2012) 1-9.

[14] He, G., H. Jia, L. Guo and P. Liu, Adaptive sampling strategy for freeform surface based on CAD model, *Advanced Materials Research*, **542-543** (2012) 541-544.

[15] Yu, M., Y. Zhang, Y. Li and D. Zhang, Adaptive sampling method for inspection planning on CMM for freeform surfaces, *International Journal of Advanced Manufacturing Technology*, **67** (2013) 1967-1975.

[16] ElKott, D. F. and S. C. Veldhuis, Isoparametric line sampling for the inspection planning of sculptured surfaces, *Computer Aided Design*, **37** (2005) 189-200.

[17] Zhou, A., J. Guo and W. Shao, Automated inspection planning of freeform surfaces for manufacturing applications, *IEEE Int. Conference on Mechatronics and Automation*, Beijing, (2011) 2264-2269.

[18] Rajamohan, G. and M. S. Shunmugam, Strategies for measurement of sculptured surfaces using coordinate measuring machines in continuous scanning mode, *Int. Journal of Mechatronics and Manufacturing Systems*, **6** (2013) 335-350.

[19] Rajamohan, G. and M. S. Shunmugam, Adaptive sampling strategies for measurement of sculptured surfaces using coordinate measuring machines in continuous scanning mode, *International Journal of Precision Technology*, **4** (2014) 3-18.

[20] Piegl, L. and W. Tiller, *The NURBS Book*, Springer-Verlag, Berlin (1997).

[21] Gray, A., *Modern Differential Geometry of Curves and Surfaces with Mathematica*, 2nd Ed., CRC Press, Boca Raton, FL. (1997).

[22] Rogers, D. F. and J. A. Adams, *Mathematical Elements for Computer Graphics*, Tata McGraw-Hill Publishing Co., New Delhi. (2002).

[23] Park, H., B-spline surface fitting based on adaptive knot placement using dominant columns, *Computer Aided Design*, **43** (2011) 258-264.

[24] Piegl, L. and W. Tiller, Surface approximation to scanned data, *The Visual Computer*, **16** (2000) 386-395.

[25] Wang, J., X. Jiang, L. A. Blunt, R. K. Leach, and P. J. Scott, Efficiency of adaptive sampling in surface texture measurement for structured surfaces. *13th Int. Conference on Metrology and Properties of Engineering Surfaces, Journal of Physics: Conference Series*, **311** (2011) 1-7.